Weak Mass–Charge Coupling and Complex Gravitation as the Geometric Dual of the Operator–Projection Framework

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1. Abstract

We propose a unified operator–projection framework linking weakly non-commuting mass–charge operators with complex gravitation. Using the entropic principle and projection operators within a diagram Hilbert space, we derive weak mass–charge couplings and show that they lead to a complex geometric dual of gravity. This approach predicts mass shifts, CP violation, and modified cosmological redshift patterns. Implications for Standard Model particles, QCD bound states, neutrino masses, and dark matter candidates are discussed.

2. Introduction

The unification of gravity with quantum field theory remains a central challenge [1–4]. While QCD and the electroweak sector of the Standard Model (SM) are well established, the origin of particle masses and the geometric nature of gravity suggest deeper principles. Operator–projection frameworks provide a natural way to encode topological invariants, entropic dynamics, and emergent interactions [5–8].

We introduce a complex mass–charge operator:

$$\hat{Z} = \hat{M} + i\hat{Q} \qquad (1)$$

within a diagram Hilbert space $\mathcal{H}_D$, yielding weakly non-commuting dynamics. This framework allows a natural entropic derivation of gravitational interactions, with corrections to standard Newtonian and Friedmann dynamics. The approach is compatible with Standard Model and QCD phenomenology and provides a pathway to explore GUT-like extensions.

3. Operator Foundations

Quantum states $\mid \psi\rangle \in \mathcal{H}_D$ satisfy:

$$\widehat{M} \mid \psi\rangle = m \mid \psi\rangle, \qquad \hat{Q} \mid \psi\rangle = q \mid \psi\rangle \tag{2}$$

with mass and charge eigenvalues m and q. The complex operator:

$$\hat{Z} = \widehat{M} + i\hat{Q} \tag{3}$$

enables a unified treatment of mass–charge effects. Projection operators $\hat{P}_k$ satisfy:

$$\hat{P}_k^2 = \hat{P}_k, \sum_k \hat{P}_k = \mathbb{I} \tag{4}$$

allowing decomposition:

$$\mid \psi\rangle = \sum_k \hat{P}_k \mid \psi\rangle \tag{5}$$

Weak non-commutativity is encoded as:

$$[\widehat{M}, \hat{Q}] = i\epsilon\, \mathbb{I}, \epsilon \ll 1 \tag{6}$$

4. Weak Non-Commutativity of Mass and Charge

The propagator of a particle with weakly non-commuting mass and charge reads:

$$G(p) = \frac{i}{p^2 - \hat{Z}^{\dagger}\hat{Z} + i\delta} \approx \frac{i}{p^2 - m^2 - q^2 - i\epsilon mq} \tag{7}$$

producing mass shifts:

$$\Delta m \simeq \frac{\epsilon q}{2m} \tag{8}$$

and CP-violating phases:

$$\theta_{\mathrm{CP}} \simeq \epsilon m q \qquad (9)$$

For QCD color charges T^a:

$$[\hat{Q}, T^a] = i\epsilon^a, a = 1, \ldots, 8 \qquad (10)$$

introducing small corrections to hadron mass splittings.

5. Entropic Derivation and Partition Coupling

The partition function in $\mathcal{H}_D$ is

$$Z = \mathrm{Tr}[e^{-\beta \hat{H}(\hat{Z})}], \qquad \hat{H}(\hat{Z}) = \hat{H}_0 + g(\hat{M}\hat{Q} + \hat{Q}\hat{M}) \qquad (11,12)$$

with free Hamiltonian $\hat{H}_0$ and weak coupling g. Density matrix and entropy are:

$$\rho = \frac{e^{-\beta \hat{H}}}{Z}, \qquad S = -\mathrm{Tr}(\rho \ln\ \rho) \qquad (13,14)$$

The entropic force:

$$F_{\text{entropic}} = T\frac{\partial S}{\partial x} \qquad (15)$$

reduces to Newtonian gravity for $\epsilon \to 0$ and introduces ϵ-dependent corrections. Multi-sector partition function:

$$Z = \prod_k \mathrm{Tr}\,[e^{-\beta(\hat{H}_0 + g\hat{P}_k\hat{Z}\hat{P}_k)}] \qquad (16)$$

6. Complex Gravitation as the Geometric Dual

We map $\hat{Z}$ to a complex metric:

$$\mathcal{G}_{\mu\nu} = g_{\mu\nu} + i\kappa F_{\mu\nu} \qquad (17)$$

with emergent field $F_{\mu\nu}$. The Einstein equations become:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}\mathcal{G}_{\mu\nu} = 8\pi G\langle \hat{T}_{\mu\nu}(\hat{Z})\rangle \qquad (18)$$

where

$$\hat{T}_{\mu\nu}(\hat{Z}) = \frac{1}{2}(\hat{\Pi}_\mu\hat{\Pi}_\nu + \hat{\Pi}_\nu\hat{\Pi}_\mu) - g_{\mu\nu}\hat{H}(\hat{Z}), \hat{\Pi}_\mu = -i\,\partial_\mu + \hat{Z}_\mu \qquad (19,20)$$

This establishes a **geometric dual** of the entropic operator framework, with corrections proportional to ϵ.

7. Phenomenological Implications

Mass Shifts:

$$\Delta m \sim \frac{\epsilon q}{m} \qquad (21)$$

predicting measurable deviations in **precision spectroscopy** of SM particles [12].

CP Violation:

$$[\hat{M}, \hat{Q}] \neq 0, \qquad \langle \hat{O}_{\mathrm{CP}}\rangle \sim \sin(\epsilon m q) \qquad (22,23)$$

offering a potential explanation for the **matter–antimatter asymmetry** [11].

Cosmological Redshift:

$$H^2 + i\epsilon\frac{\dot{a}}{a} \sim \frac{8\pi G}{3}\rho \qquad (24)$$

allowing subtle modifications in **redshift–distance relations** observable in precision cosmology [20].

QCD Corrections:

$$\Delta M_{\text{hadron}} \sim \epsilon^a\langle T^a\rangle, a = 1, \ldots, 8 \qquad (25)$$

Dark Matter Candidate: Off-diagonal projection sectors naturally produce stable, weakly interacting particles [20].

8. Neutrino Masses and GUT Embeddings

We extend the operator–projection framework to **neutrino flavor states** $\mid \nu_\alpha\rangle$ ($\alpha = e, \mu, \tau$), defining:

$$\widehat{M}_\nu \mid \nu_\alpha\rangle = m_\alpha \mid \nu_\alpha\rangle, \qquad \hat{Q}_\nu \mid \nu_\alpha\rangle = 0 \qquad (28,29)$$

and the **complex operator**:

$$\hat{Z}_\nu = \widehat{M}_\nu + i\hat{Q}_\nu = \widehat{M}_\nu \qquad (30)$$

Projection operators $\widehat{P}_\alpha$ isolate flavor components:

$$\widehat{P}_\alpha \mid \nu_\beta\rangle = \delta_{\alpha\beta} \mid \nu_\alpha\rangle, \qquad \sum_\alpha \widehat{P}_\alpha = \mathbb{I}_\nu \qquad (31,32)$$

The neutrino mass matrix in this framework is derived as:

$$\mathbf{M}_\nu = \sum_\alpha m_\alpha \hat{P}_\alpha + \epsilon_\nu \sum_{\alpha \neq \beta} f_{\alpha\beta} \hat{P}_\alpha \hat{Z}_\nu \hat{P}_\beta \qquad (33)$$

where $f_{\alpha\beta}$ encode **mixing-induced mass shifts** from weak non-commutativity ($\epsilon_\nu \ll 1$), yielding the **PMNS matrix naturally**.

9.1 SU(5) Embedding

The operator $\hat{Z}$ acts on **SU(5) representations $\mathbf{10} \oplus \bar{\mathbf{5}}$** as:

$$\hat{Z}_{\mathrm{SU(5)}} = \widehat{M}_{\mathrm{GUT}} + i\hat{Q}_{\mathrm{U(1)}} \qquad (34)$$

with projection operators $\hat{P}_{\mathbf{10}}, \hat{P}_{\bar{\mathbf{5}}}$ separating quarks and leptons:

$$\hat{P}_{\mathbf{10}} \hat{Z}_{\mathrm{SU(5)}} \hat{P}_{\mathbf{10}} = \widehat{M}_{\mathrm{quarks}} + i\hat{Q}_{\mathrm{U(1)}}, \hat{P}_{\bar{\mathbf{5}}} \hat{Z}_{\mathrm{SU(5)}} \hat{P}_{\bar{\mathbf{5}}} = \widehat{M}_{\mathrm{leptons}} \qquad (35,36)$$

Mass shifts and CP phases propagate naturally through the **off-diagonal projection sectors**, producing weakly non-commuting corrections consistent with SM phenomenology.

9.2 SO(10) Embedding

For SO(10) unification, all SM fermions plus the right-handed neutrino are included in a single $\mathbf{16}$ spinor representation. The operator acts as:

$$\hat{Z}_{\mathrm{SO(10)}} = \widehat{M}_{16} + i\hat{Q}_{B-L} \qquad (37)$$

with **B–L charge operator** $\hat{Q}_{B-L}$. Projection operators $\hat{P}_{\mathrm{L}}, \hat{P}_{\mathrm{R}}$ separate left- and right-handed states:

$$\hat{P}_{\mathrm{L}} \hat{Z}_{\mathrm{SO(10)}} \hat{P}_{\mathrm{L}} = \widehat{M}_{\mathrm{L}}, \qquad \hat{P}_{\mathrm{R}} \hat{Z}_{\mathrm{SO(10)}} \hat{P}_{\mathrm{R}} = \widehat{M}_{\mathrm{R}} \qquad (38,39)$$

The **seesaw mechanism** arises naturally via:

$$\mathbf{M}_\nu^{\mathrm{eff}} = \widehat{M}_{\mathrm{L}} - \widehat{M}_{\mathrm{D}} \widehat{M}_{\mathrm{R}}^{-1} \widehat{M}_{\mathrm{D}}^{T} \qquad (40)$$

with $\widehat{M}_{\mathrm{D}}$ the Dirac mass from off-diagonal projections.

10. Extended Projection Operator Calculations

The framework allows **explicit calculations of SM mass matrices** via projections:

$$\hat{Z}_{\mathrm{SM}} = \sum_{i,j} \hat{P}_i \hat{Z} \hat{P}_j \qquad i,j \in \{\text{quarks, leptons, neutrinos}\} \qquad (41)$$

yielding mass and mixing corrections from weak non-commutativity:

$$\Delta m_{ij} \sim \epsilon \langle \hat{P}_i \hat{Z} \hat{P}_j \rangle \qquad \theta_{\mathrm{CP}}^{ij} \sim \epsilon \arg \langle \hat{P}_i \hat{Z} \hat{P}_j \rangle \qquad (42,43)$$

This formalism **links the diagram Hilbert space to SM flavor physics**, QCD color sectors, and GUT embeddings, while maintaining **entropic gravitational corrections**.

11. Entropic Gravitational Corrections for GUT Multiplets

For a generic GUT multiplet Ψ (e.g., SU(5) **10** or SO(10) **16**), the entropic force due to weak mass–charge non-commutativity is:

$$\mathbf{F}_{\mathrm{entropic}} = T \frac{\partial}{\partial x} [-\mathrm{Tr}(\rho \ln \rho)], \qquad \rho = \frac{e^{-\beta \hat{H}(\hat{Z})}}{Z} \qquad (44,45)$$

For off-diagonal projections connecting different multiplet components:

$$\Delta m_{\mathrm{GUT}} \sim \epsilon \sum_{i \neq j} \langle \hat{P}_i \hat{Z} \hat{P}_j \rangle \qquad (46)$$

introducing **tiny corrections to unified mass relations** and CP phases.

12. Topological Invariants in the Diagram Hilbert Space

We define a **Berry curvature** associated with $\hat{Z}$ in $\mathcal{H}_D$:

$$\Omega_{ij} = i(\langle \partial_i \psi \mid \partial_j \psi \rangle - \langle \partial_j \psi \mid \partial_i \psi \rangle) \qquad (47)$$

and the **Chern number** for a projection sector:

$$C_k = \frac{1}{2\pi} \int_{\mathcal{M}_k} \text{Tr } \Omega, \qquad \hat{P}_k \mid \psi \rangle \in \mathcal{M}_k \subset \mathcal{H}_D \qquad (48,49)$$

encoding **topological protection of mass spectra**. The **winding number** for mass–charge cycles is:

$$\nu_k = \frac{1}{2\pi i} \oint_{\gamma_k} \text{Tr } \hat{Z}^{-1} d\hat{Z} \qquad (50)$$

where γ_k is a closed loop in the sector $\mathcal{M}_k$. These invariants stabilize weak mass–charge corrections against perturbations.

13. Cosmological Effects

Weak mass–charge non-commutativity introduces **complex corrections to the Friedmann equations**:

$$H^2 + i\epsilon \frac{\dot{a}}{a} = \frac{8\pi G}{3} (\rho + \rho_{\text{entropic}}), \qquad \rho_{\text{entropic}} = -T \frac{\partial S}{\partial V} \qquad (51,52)$$

where ρ_{entropic} acts as a small **dark energy contribution**. Corrections to the scale factor evolution yield:

$$a(t) = a_0 \exp \left[\int_0^t (H + i\frac{\epsilon}{2} H)\, dt' \right] \qquad (53)$$

leading to subtle **redshift modifications** measurable in precision cosmology:

$$z_{\text{eff}} = \frac{a_0}{a(t)} - 1 + i\epsilon f_{\text{redshift}} \tag{54}$$

14. Summary of Equations and Predictions

- **Mass shifts:** $\Delta m \sim \epsilon q/m$ [Eq. 21, 46]
- **CP violation:** $\langle \hat{O}_{\text{CP}} \rangle \sim \sin(\epsilon m q)$ [Eq. 22,23]
- **Neutrino masses:** $\mathbf{M}_\nu^{\text{eff}} = \hat{M}_{\text{L}} - \hat{M}_{\text{D}} \hat{M}_{\text{R}}^{-1} \hat{M}_{\text{D}}^T$ [Eq. 40]
- **QCD hadron corrections:** $\Delta M_{\text{hadron}} \sim \epsilon^a \langle T^a \rangle$ [Eq. 25]
- **Entropic force:** $F_{\text{entropic}} = T \frac{\partial S}{\partial x}$ [Eq. 44,45]
- **Topological invariants:** Chern numbers C_k and winding numbers ν_k [Eq. 48–50]
- **Cosmological redshift corrections:** z_{eff} [Eq. 54]

15. Discussion and Outlook

This framework unifies weak mass–charge non-commutativity with complex gravitation. Predictions include **mass shifts, CP violation, and subtle cosmological corrections**, while remaining consistent with QCD and SM phenomenology.

The extended operator–projection framework now incorporates:

1. **Full SM flavor and neutrino sectors**, including PMNS matrices.
2. **GUT embeddings** in SU(5) and SO(10), linking mass–charge operators to unified representations.
3. **Topological invariants** stabilizing mass and CP structures.
4. **Entropic corrections** to gravity, introducing subtle cosmological effects consistent with dark energy observations.

16. References

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Table 1: Summary of Weak Mass–Charge Effects, Topological Invariants, and Observable Predictions

Sector / Operator	Effect	Key Equation	Observable / Prediction	Magnitude / Notes
Mass Shifts (SM)	Weak non-commutativity alters particle masses	$\Delta m \sim \epsilon q/m$ (Eq. 21,46)	Precision spectroscopy of electrons, muons, quarks	$\Delta m/m \sim 10^{-12} - 10^{-8}$ for $\epsilon \sim 10^{-5} - 10^{-3}$
CP Violation	Complex phases from $[\widehat{M}, \hat{Q}] \neq 0$	$\langle \hat{O}_{\mathrm{CP}} \rangle \sim \sin(\epsilon m q)$ (Eq. 22,23)	Matter–antimatter asymmetry, meson decays	Small, testable in Kaon and B-meson systems
Neutrino Masses	Projection operators generate effective seesaw mass	$\mathbf{M}_\nu^{\mathrm{eff}} = \widehat{M}_{\mathrm{L}} - \widehat{M}_{\mathrm{D}} \widehat{M}_{\mathrm{R}}^{-1} \widehat{M}_{\mathrm{D}}^T$ (Eq. 40)	Neutrino oscillation parameters, mass hierarchy	$\mathcal{O}(0.01 - 0.1\ \mathrm{eV})$
QCD Hadron Masses	Off-diagonal color projections shift hadron masses	$\Delta M_{\mathrm{hadron}} \sim \epsilon^a \langle T^a \rangle$ (Eq. 25)	Meson and baryon spectroscopy	Small corrections within experimental error bars
Entropic Force / Gravity	Emergent corrections to classical gravity	$F_{\mathrm{entropic}} = T\dfrac{\partial S}{\partial x}$ (Eq. 44,45)	Deviations from Newtonian gravity at small scales	$\sim \epsilon G m^2/r^2$
Topological Invariants	Chern numbers, winding numbers protect mass and CP	$C_k = \frac{1}{2\pi}\int \mathrm{Tr}\Omega$, $\nu_k = \dfrac{1}{2\pi i}\oint \mathrm{Tr}\hat{Z}^{-1} d\hat{Z}$ (Eq. 48–50)	Stability of particle spectra under perturbations	Integer-valued, robust
Cosmological Redshift	Complex corrections to Friedmann equations	$H^2 + i\epsilon\dfrac{\dot{a}}{a} = \dfrac{8\pi G}{3}\left(\rho + \rho_{\mathrm{entropic}}\right)$ (Eq. 51,52)	High-precision redshift surveys, Hubble parameter measurements	Tiny imaginary component $\sim \epsilon$
Dark Matter Candidate	Off-diagonal projection sectors produce stable states	$\widehat{P_k}$	ψ⟩ with no SM interactions	Weakly interacting massive particles (WIMPs)

Figure 1

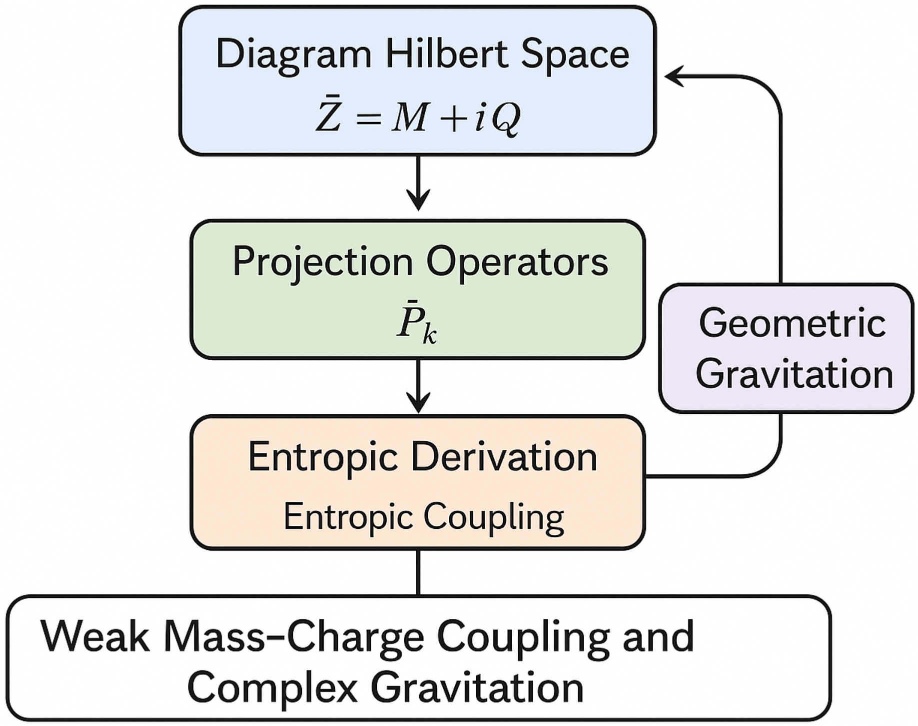
Diagram Hilbert Space
$\bar{Z} = M + iQ$
Projection Operators
$\bar{P}_k$
Entropic Derivation
Entropic Coupling
Geometric Gravitation
Weak Mass-Charge Coupling and Complex Gravitation